Tribhuvan University

Institute of Science and Technology

Model Question Paper

Bachelor Level/ First Year/ Second Semester/ Science — Full Marks: 80

Computer Science and Information Technology (MTH 155) — Pass Marks: 32

(Linear Algebra) — Time: 3 hours.

Candidates are required to give their answers in their own words as for as practicable.

The figures in the margin indicate full marks.

Attempt all questions:

Section A: Short Answer Questions. **(10x2=20)**

1. If one row in an echelon form of an augmented matrix is $[0 \quad 0 \quad 0 \quad 5 \quad 0]$, then the associated linear system is inconsistent. State whether the statement is true or false. Give reasons.
2. Let $\vec{u} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, show that $\begin{bmatrix} h \\ k \end{bmatrix}$ is in the span $\{u, v\}$ for all h and k.
3. Let $A = \begin{pmatrix} 2 & 5 \\ -3 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & -5 \\ 3 & k \end{pmatrix}$, what value(s) of k, if any, will make AB = BA?
4. If $A = \begin{bmatrix} 6 & 1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$, then show that det(AB) = det(A) . det(B).
5. Using Cramer's rule solve the following simultaneous equations:

   $5x + 7y = 3$

   $2x + 4y = 1$
6. Find a matrix A such that W = ColA.

$$W = \left\{ \begin{bmatrix} 6a - b \\ a + b \\ -7a \end{bmatrix} : a, b \ in \ R \right\}$$

7. The set $B = \{1 + t^2, t + t^2, 1 + 2t + t^2\}$ is a basis for P_2. Find the coordinate vector of $\vec{p}\,(t) = 1 + 4t + 7t^2$ relative to B.
8. Show that 7 is an eigen value of matrix $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$.
9. Let $\vec{v} = (1, -2, 2, 0)$. Find a unit vector $\vec{u}$ in the same direction as $\vec{v}$.
10. Let $\vec{v} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$ and $\vec{u} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$. Find the orthogonal projection of $\vec{v}$ onto $\vec{u}$.

Section B: Brief Answer Questions.

Attempt all questions. **(5x4=20)**

11. Find an equation involving g, h and k that makes the augmented matrix correspond to the consistent system.

$$\begin{bmatrix} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{bmatrix}$$

12. Find the 3x3 matrix and corresponds to the composite transformation of a scaling by 0.3, a rotation of 90^0, and finally a translation that adds (9, -5, 2) to each point of a figure.

13. Find the inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix}$, if it exists.

OR

Find a least square solution of Ax = b for

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}, \qquad b = \begin{bmatrix} -3 \\ -1 \\ 0 \\ 2 \\ 5 \\ 1 \end{bmatrix}$$

14. Define null space of an m x n matrix A. prove that null space of an m x n matrix A is subspace of R^n.
15. Define Characteristics equation. Find the Characteristics equation of

$$A = \begin{bmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & -8 & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Section C: LONG ANSWER QUESTIONS.

Attempt all questions. **(5 x 8 = 40)**

16. Define linearly independent and linearly dependent set in R^n. Let $v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$, $v_3 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$
    a. Determine if the set $\{v_1, v_2, v_3\}$ is linearly independent.
    b. If possible find a linearly dependent relating among v_1, v_2, v_3.

OR

Let $A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}, u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, b = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}, c = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$ and a transformation $T : R^2 \rightarrow R^3$ defined by T(x) = Ax,
    a. Find T(u).
    b. Find an x in R^2 whose image under T is b.
    c. Is there more than one x whose image under T is b.
    d. Determine if c is the in the range of the transformation T.
17. Define block upper triangular. Assume that A11 is p x p, A22 is q x q, and A is invertible. Find a formula for A^{-1}.
18. Define basis and dimension of vector space. Find the basis and dimension of the subspace$H = \left\{ \begin{bmatrix} a - 3b + 6c \\ 5a + 4d \\ b - 2c - d \\ 5d \end{bmatrix} : a, b, c, d \in \mathcal{R} \right\}$
19. Diagonalize the matrix

$$A = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 1 & 4 & -3 & 0 \\ -1 & -2 & 0 & -3 \end{bmatrix}$$

Find an invertible matrix P and diagonal matrix D, such that $A = PDP^{-1}$.

20. What do you understand by Gram-Schmidt process? Let $x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $x_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $x_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$.

Then $\{x_1, \ x_2, \ x_3\}$. Using Gram-Schmidt process construct an orthogonal basis for W.

OR

Let u and v are non-zero vectors in $\mathcal{R}^2$ and $\mathcal{R}^3$ and α be the angle between u and v then prove that $u.v = \|u\|\|v\|cos\alpha$. Represent it geometrically.

Tribhuvan University

Institute of Science and Technology

2065

Bachelor Level/ First Year/ Second Semester/ Science — Full Marks: 80

Computer Science and Information Technology (MTH 155) — Pass Marks: 32

(Linear Algebra) — Time: 3 hours.

Candidates are required to give their answer in their own words as for as practicable.

The figures in the margin indicate full marks.

Attempt all the questions:

Group A (10x2=20)

1. Illustrate by example that a system of linear equations has either exactly one solution or infinitely many solutions.

2. What is a linear transformation invertible?

3. Solve the system.
   $3x_1 + 4x_2 = 3$
   $5x_1 + 6x_2 = 7$
   by using the inverse of the matrix $A = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$.

4. State the numerical importance of determinant calculation by row operation.

5. State Cramer's Rule for an invertible n x n matrix A and vector $b \in \boldsymbol{R}^n$ to solve the system $Ax = b$. Is this method efficient from computational point of view?

6. Determine if $\{v_1, v_2, v_3\}$ is a basis for $\boldsymbol{R}^3$, where $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix},\ v_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

7. Determine if $W = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$ is a $Nul(A)$ for $A = \begin{bmatrix} 3 & -5 & -3 \\ 6 & -2 & 0 \\ -8 & 4 & 1 \end{bmatrix}$.

8. Show that 7 is an eigenvalue of $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$.
9. If $S = \{u_1, \dots \dots \dots \dots, u_p\}$ is an orthogonal set of nonzero vectors in $\boldsymbol{R}^2$, then S is linearly independent and hence is a basis for the subspace spanned by S.

10. Let $W = span\{x_1, x_2\}$ where $x_1 = \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}$ and $x_2 = \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$. Then construct orthogonal basis for W.

Group B (5x4=20)

11. Determine if the given set is linearly dependent:

(a) $\begin{bmatrix}1\\7\\6\end{bmatrix}, \begin{bmatrix}2\\0\\9\end{bmatrix}, \begin{bmatrix}3\\1\\5\end{bmatrix}, \begin{bmatrix}4\\1\\8\end{bmatrix}$ (b) $\begin{bmatrix}-2\\4\\6\\10\end{bmatrix}, \begin{bmatrix}3\\-6\\-9\\15\end{bmatrix}$

12. Find the 3x3 matrix that corresponds to the composite transformation of a scaling by 0.3, a rotation of 90^0, and finally a translation that adds (-0.5, 2) to each point of a figure.

OR

Describe the Leontief Input-Output model for certain economy and derive formula for $(I\text{-}C)^{-1}$, where the symbols have their usual meanings.

13. Find the coordinate vector $[x]_B$ of x relative to the given basis $B=\{b_1, b_2\}$, where $b_1 = \begin{bmatrix}1\\-3\end{bmatrix}$, $b_2 = \begin{bmatrix}2\\-5\end{bmatrix}$, $x = \begin{bmatrix}-2\\1\end{bmatrix}$.

14. Let $A = \begin{bmatrix}4 & -9\\4 & 8\end{bmatrix}$, $b_1 = \begin{bmatrix}3\\2\end{bmatrix}$, $b_2 = \begin{bmatrix}2\\1\end{bmatrix}$ and basis $B = \{b_1,\ b_2\}$. Find the B-matrix for the transformation $x \rightarrow Ax$ with $P = \{b_1,\ b_2\}$.

15. Let u and v be nonzero vectors in $\boldsymbol{R}^3$ and the angle between them be ϕ. Then prove that
$$u.v = \|u\|\ \|v\| \cos\phi,$$
where the symbols have their usual meanings.

Group C (5x8=40)

16. Let $T:R^n \rightarrow R^m$ be a linear transformation. Then T is one to one if and only if the equation $T(x) = 0$ has only the trivial solution, prove the statement.

OR

Let $A = \begin{bmatrix}1 & -3\\3 & 5\\-1 & 7\end{bmatrix}$, $u = \begin{bmatrix}2\\-1\end{bmatrix}$, $b = \begin{bmatrix}3\\2\\-5\end{bmatrix}$, $c = \begin{bmatrix}3\\2\\5\end{bmatrix}$

and define $T : \mathbf{R}^2 \rightarrow \mathbf{R}^3$ by $T(x) = Ax$. Then

(a) Find $T(u)$.
(b) Find an $x \in \mathbf{R}^2$ whose image under T is b.
(c) Is there more than one x whose image under T is b.
(d) Determine if c is the range of T.

17. Compute the multiplication of partitioned matrices for

$$A = \left[\begin{array}{ccc|cc} 2 & -3 & 1 & 0 & -4 \\ 1 & 5 & -2 & 3 & -1 \\ \hline 0 & 4 & -2 & 7 & -1 \end{array}\right] \quad \text{and} \quad B = \left[\begin{array}{cc} 6 & 4 \\ -2 & 1 \\ -3 & 7 \\ \hline -1 & 3 \\ 5 & 2 \end{array}\right]$$

18. What do you mean by change of basis in $\mathbf{R}^n$? Let $b_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$, $b_2 = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$, $c_1 = \begin{bmatrix} -7 \\ 9 \end{bmatrix}$, $c_2 = \begin{bmatrix} -5 \\ 7 \end{bmatrix}$, and consider the bases for $\mathbf{R}^2$ given by $B = \{b_1, b_2\}$ and $C = \{c_1, c_2\}$.
    (a) Find the change of coordinate matrix from C to B.
    (b) Find the change of coordinate matrix from B to C.

OR

Define vector space, subspace, basis of a vector space with suitable examples. What do you mean by linearly independent set and linearly dependent set of vectors?

19. Diagonalize the matrix $A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$, if possible.

20. Find the equation $y = \beta_0 + \beta_1 x$ of the least squares line that best fits the data points (2, 1), (5, 2), (7, 3), (8, 3). What do you mean by least squares line?

Tribhuvan University

Institute of Science and Technology

2066

Bachelor Level/ First Year/ Second Semester/ Science — Full Marks: 80

Computer Science and Information Technology (MTH. 155) — Pass Marks: 32

(Linear Algebra) — Time: 3 hours.

Candidates are required to give their answers in their own words as for as practicable.

The figures in the margin indicate full marks.

Attempt all questions:

Group A (10x2=20)

1. When is a system of linear equations consistent or inconsistent?
2. Write numerical importance of partitioning matrices.
3. How do you distinguish singular and non-singular matrices?
4. If A and B are n x n matrices, then verity with an example that det (AB) = det (A) det (B).
5. Calculate the area of the parallelogram determined by the columns of $A = \begin{bmatrix} 2 & 6 \\ 5 & 1 \end{bmatrix}$.
6. Determine if $w = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$ is in Nul (A), where, $A = \begin{bmatrix} 3 & -5 & -3 \\ 6 & -2 & 0 \\ -8 & 4 & 1 \end{bmatrix}$.
7. Determine if $\{v_1, v_2, v_3\}$ is a basis for $\boldsymbol{R}^3$, where $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$
8. Find the characteristics polynomial and the eigen values of the matrix $\begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}$.
9. Let $\vec{v} = (1, -2, 2, 0)$. Find a unit vector $\vec{u}$ in the same direction as $\vec{v}$.
10. Let $\{u_1, \ldots\ldots, u_p\}$ be an orthogonal basis for a subspace W of $\boldsymbol{R}^n$. Then prove that for each y, ϵ, W, the weights in $y = c_1u_1 + \cdots + c_pu_p$ are given by

$$c_j = \frac{y.u_j}{u_j.u_j} \qquad (\ j = 1, \ldots\ldots, p)$$

Group B (5x4=20)

11. Prove that any set $\{v_1, \ldots\ldots, v_p\}$ in R^n is linearly dependent if $p > n$.
12. Consider the Leontief input-output model equation $x = cx + d$, where the consumption matrix is $C = \begin{bmatrix} .50 & .40 & .20 \\ .20 & .30 & .10 \\ .10 & .10 & .30 \end{bmatrix}$.

    Suppose the final demand is 50 units for manufacturing 30 units for agriculture, 20 units for services. Find the production level x that will satisfy this demand.
13. What do you mean by basis of a vector space? Find the basis for the row space of

$$A = \begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix}$$

OR

State and prove the unique representation theorem for coordinate system.

14. What do you mean by eigen values, eigen vectors and characteristic polynomial of a matrix? Explain with suitable examples.
15. Define a Gram-Schmidt process. Let $W = span\{x_1, x_2\}$, where
$x_1 = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$, $x_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$.
Then construct an orthogonal basis $\{v_1, v_2\}$ for w.

Group C (5x8=40)

16. Given the matrix $\begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & -15 \end{bmatrix}$, discuss the forward phase and backward phase of the row reduction algorithm.
17. Find the inverse of $\begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$, if it exists, by using elementary row reduce the augmented matrix.
18. What do you mean by change of basis in $\boldsymbol{R}^n$? Let $b_1 = \begin{bmatrix} -9 \\ 1 \end{bmatrix}, b_2 = \begin{bmatrix} -5 \\ -1 \end{bmatrix}, c_1 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}, c_2 = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$, and consider the bases for $\boldsymbol{R}^2$ given by $B = \{b_1, b_2\}$ and $C = \{c_1, c_2\}$. Find the change of coordinated matrix from B to C.
19. Diagonalize the matrix $\begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{bmatrix}$, if possible.

OR

Find the eigen values of $A = \begin{bmatrix} 0.50 & -0.60 \\ 0.75 & 1.1 \end{bmatrix}$, and find the basis for each eigen space.

20. Find a least-squares solution for $Ax = b$ with
$A = \begin{bmatrix} 1 & -6 \\ 1 & -2 \\ 1 & 1 \\ 1 & 7 \end{bmatrix}, b = \begin{bmatrix} -1 \\ 2 \\ 1 \\ 6 \end{bmatrix}$.
What do you mean by least-squares problem?

OR

Define a least-squares solution of $Ax = b$, prove that the set of least squares solutions of $Ax = b$ coincides with the nonempty set of solutions of the normal equations $A^T Ax = A^T b$.

Tribhuvan University

Institute of Science and Technology

2067

Bachelor Level/ First Year/ Second Semester/ Science — Full Marks: 80

Computer Science and Information Technology (MTH 155) — Pass Marks: 32

(Linear Algebra) — Time: 3 hours.

Candidates are required to give their answers in their own words as for as practicable.

The figures in the margin indicate full marks.

Attempt all questions:

Group A (10x2=20)

1. Illustrate by an example that a system of linear equations has either no solution or exactly one solution.
2. Define singular and nonsingular matrices.
3. Using the Invertible matrix Theorem or otherwise, show that
$$A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 1 & -2 \\ -5 & -1 & 9 \end{bmatrix}$$
is invertible.
4. What is numerical drawback of the direct calculation of the determinants?
5. Verify with an example that det (AB) = det (A) det (B) for any n x n matrices A and B.
6. Find a matrix A such that $w = col(A)$.
$$w = \left\{ \begin{bmatrix} 6a - b \\ a + b \\ -7a \end{bmatrix} : a, b \in R \right\}.$$
7. Define subspace of a vector space with an example.
8. Are the vectors;
$u = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$ and $v = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ eigenvectors of $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$?
9. Find the distance between vectors u = (7, 1) and v = (3, 2). Define the distance between two vectors in R^n.
10. Let w = span {x_1, x_2}, where $x_1 = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$, $x_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$.
Then construct orthogonal basis for w.

Group B (5x4=20)

11. If a set s = {v_1, v_2, … …, v_p} in R^n contains the zero vector, then prove that the set is linearly dependent. Determine if the set
$$\begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 1 \\ 8 \end{bmatrix}$$
is linearly dependent.
12. Given the Leontief input-output model x = Cx + d, where the symbols have their usual meanings, consider any economy whose consumption matrix is given by
$$C = \begin{bmatrix} .50 & .40 & .20 \\ .20 & .30 & .10 \\ .10 & .10 & .30 \end{bmatrix}$$

Suppose the final demand is 50 units for manufacturing 30 units for agriculture, 20 units for services. Find the production level x that will satisfy this demand.

13. Define rank of a matrix and state Rank Theorem. If A is a 7 x 9 matrix with a two-dimensional null space, find the rank of A.
14. Determine the eigen values and eigenvectors of $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ in complex numbers.

OR

Let $A = \begin{bmatrix} 4 & -9 \\ 4 & 8 \end{bmatrix}$, $b_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, $b_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and basis $B = \{b_1, b_2\}$.

Find the B-matrix for the transformation x→ x with P = [b_1, b_2].

15. Let u and v be nonzero vectors in R^2 and the angle between them be θ then prove that
u.v = ||u|| ||v|| cos θ,
where the symbols have their usual meanings.
16. Determine if the following homogeneous system has a nontrivial solution. Then describe the solution set.
$3x_1 + 5x_2 - 4x_3 = 0,\ -3x_1 - 2x_2 + 4x_3 = 0,\ 6x_1 + x_2 - 8x_3 = 0.$
17. An n x n matrix A is invertible if and only if A is row equivalent to I_n , and in this case, any sequence of elementary row operations that reduces A to I_n also transform $I_{n\,x\,m}$ into A^{-1}.
Use this statement to find the inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix}$, if exists.
18. What do you mean by basis change? Consider two bases B = {b_1, b_2} and c = {c_1, c_2} for a vector space V, such that $b_1 = 4c_1 + c_2$ and $b_2 = 6c_1 + c_2$. Suppose $[x]_B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ i.e., x = 3b_1 + b_2. Find $[x]_C$.

OR

Define basis of a subspace of a vector space.

Let $v_1 = \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}, v_2 = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 6 \\ 16 \\ -5 \end{bmatrix}$, where $v_3 = 5v_1 + 3v_2$, and let H = span $\{v_1, v_2, v_3\}$.

Show that span $\{v_1, v_2, v_3\}$ = span $\{v_1, v_2\}$ and find a basis for the subspace H.

19. Diagonalize the matrix $A = \begin{bmatrix} 2 & 4 & 3 \\ -4 & -6 & -3 \\ 3 & 3 & 1 \end{bmatrix}$, if possible.
20. What do you mean by least-squares lines? Find the equation $y = \beta_0 + \beta_1 x$ of the least-squares line that fits the data points (2, 1), (5, 2), (7, 3), (8, 3).

OR

Find the least-squares solution of Ax = b for

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{bmatrix},\ b = \begin{bmatrix} 3 \\ 5 \\ 7 \\ -3 \end{bmatrix}.$$